Two-Scale FEM Approach in the Dynamic Response of a Heterogeneous Material

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Consider a heterogeneous material composed of grains and binder as shown in Fig. 1 having approximately constant material properties relative to a domain larger than the grains size. In the determination of the dynamic response of such a material, a common approach is to use a homogenized model based on the average values of the material properties relative to the Representative Volume Element (RVE). In the context of the Finite Element Method (FEM) the domain is discretized in elements, each of them having constant averaged material properties and accordingly, one can determine an average response of the material. However if one's interest is in the details below the RVE size, then a fine discretization needs to be used in order to capture the local effects. This can easily lead to a large number of elements and a very small FEM time step, thus leading to a substantial increase in the time necessary for numerical simulations. To overcome these situations we propose a two-scale FEM approach, which can capture the local effects while still keeping a relatively coarse discretization in the dynamic analysis.

For a material having a local structure as shown in Fig. 1 consider a relatively coarse FEM discretization (this will be the 1st scale), similar to what is shown in Fig. 2 (a) for instance. For each element at the 1st scale, consider a second mesh, like in Fig. 2 (b) (this will be the 2nd scale), which captures the local structure. Following [Ref. 1], in order to describe the variation of a magnitude at both levels, one introduces two variables: X to describe the dependency of that magnitude at the 1st scale and y to describe its dependency at the 2nd, scale, relative to the 1st. Thus the velocity and the strain field can be written as

$$v = V(X,t) + \eta\tilde{v}(y), \quad \dot{\varepsilon} = \dot{\varepsilon}(X,t) + \dot{\tilde{\varepsilon}}(y), \tag{1}$$

where t is the time, V(X,t) and $\dot{\varepsilon}$(X,t) represent the velocity and strain rate at the 1st scale, and $\tilde{v}(y)$ and $\tilde{\varepsilon}(y)$ are the fluctuating velocities and strain rate at the 2nd scale. Introducing two FEM discretizations: a coarse one (a) corresponding to the 1st scale, and a local one (b) corresponding to the 2nd scale, as shown for example in Fig. 2, then

$$v = \Phi(X)V + \eta\phi(y)\tilde{v}, \quad \dot{\varepsilon} = B(X)V + b(y)\tilde{v}, \tag{2}$$

where $\Phi(X)$ and $\Phi(y)$ are the shape functions corresponding to the meshes at 1st and 2nd scale, and $B(X) = \partial\Phi(X) / \partial(X)$, and $b(y) = \partial\Phi(y) / \partial(y)$.

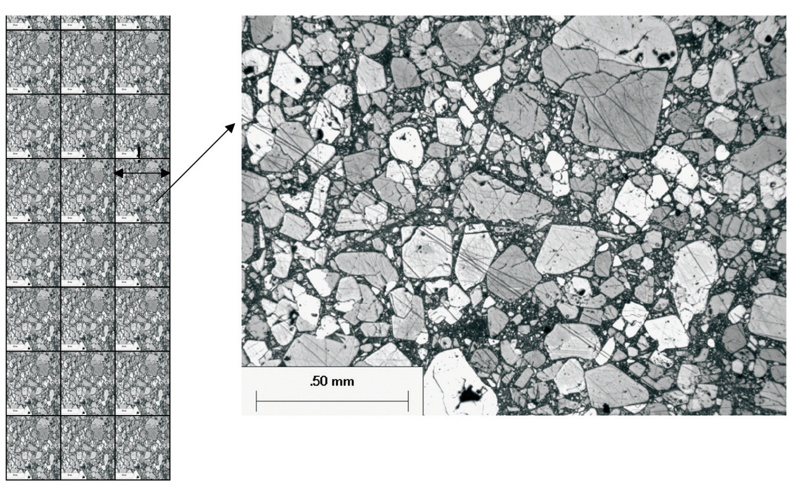

Fig. 1. *A heterogeneous material exhibiting approximately constant material properties relative to a "window" (RVE) of size "w" (a). The right side (b) shows the "window" structure (picture of a micrograph of PBX 9501).*

Using the Principle of Virtual Power [2] written in the average sense at $t + \Delta t$ (Δt being the time step) for each element at the 1st scale, and admitting the conservation of momentum while passing from the 1st to the 2nd scale, then for a constitutive law of the form $\sigma = D(\dot{\varepsilon} - \dot{\varepsilon}^{\rho})$ [3], where σ represents an objective stress rate and $\dot{\varepsilon}^{\rho}$ is the inelastic strain rate, one obtains a system having the following structure

$$\begin{bmatrix} < B^T DB > & < B^T Db > \\ < b^T DB > & < b^T Db > \end{bmatrix} \begin{pmatrix} V\Delta t \\ \tilde{v}\Delta t \end{pmatrix} = \begin{pmatrix} F_1(1^{st}) \\ F_2(2^{nd}) \end{pmatrix}. \quad (3)$$

Equation 3 represents the two-scale FEM equations. On the left side one recognizes the well-known finite element stiffness matrix form with the observation that $< B^T DB >$ represents in this case the contribution due to the first scale, $< b^T Db >$ represents the contribution due to the second scale, $< B^T Db >=< (b^T DB)^T >$ and the terms are the coupling terms between the scales. It can be shown that in Eq. 3 the inertial forces appear only at the 1st scale, i.e., in F_1 (1^{st}), and F_2 (2^{nd}) contains terms due to the inelastic strain accumulation at the 2nd scale during the current time step.

In an explicit dynamic FEM code one knows the velocities at the 1st scale $V(X,t + \Delta t)$.

Also on the boundary of the finite element of the 1st scale $\tilde{v}(y) = 0$. Thus the system Eq. 3 can be solved and the fluctuating velocities can be determined. Then the strains can be calculated using Eq. 2, and the stress distributions at the 2nd scale can be obtained by integrating the constitutive law. The feedback to the 1st scale is realized by passing the average values of those stresses, and further use them to calculate the nodal forces at the 1st scale in order to advance to the next time step.

In the above formulation the two-scale FEM approach allows us to capture local effects at the 2nd scale while preserving a relatively coarse discretization at the 1st scale. The use of conservation of momentum while passing between the two scales allows the problem to be to split in two parts. In the first part, the dynamics are solved at the 1st scale while the 2nd scale is used to determine the material response and the fluctuating fields. One may notice that these fields are determined by solving a quasistatic type of problem at each finite element of the 1st scale.

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[1] P.M. Suquet, "Elements of Homogenization for Inelastic Solid Mechanics", in *Lecture Notes in Physics. Homogenization Techniques for Composite Media*, edited by Sanchez-Palencia, E. and Zaoui A., Springer-Verlag, Vol. 272, pp. 193-278.
[2] T. Belytschko, et al., *Nonlinear Finite Element for Continua and Structures* (J. Wiley & Sons, 2004).
[3] B.E. Cements and E.M. Mas, "A Theory for Plastic-Bonded Materials with Bimodal Size Distribution of Filler Particles," *Modelling Simul. Mater. Sci. Eng*. **12**, 407-421 (2004).

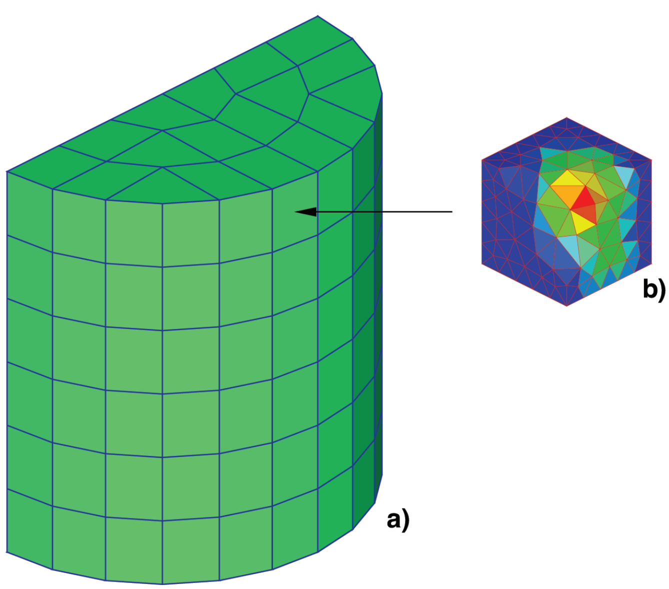

Fig. 2.
A finite element discretization at 1st scale (a) and 2nd scale (b).